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Residual stresses arising in elastoplastic materials deformed in active media under torsion

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Solutions to problems of pure elastoplastic torsion in surface-plasticized and surface-hardened rods are given. The distribution of residual tangential stresses along the rod radius is shown to have an oscillating and sign-alternating character in both cases. In surface-plasticized rods, in contrast to surface-hardened rods, the residual stresses in the external layer always have the opposite sign with respect to the active loading stresses produced by the outer torsional moment. Conclusions have been drawn about the character of the defect structure changes as well as about the conditions of sample fracture under cyclic torsional tests during the action of alternating outer force moments.

1. Introduction

The residual stresses arising in surface-modified materials after uniaxial extension, constriction or bending have been determined [1,2] using the methods of theory of elastoplasticity. The present work describes the results of a theoretical study devoted to the analysis of the residual stresses found in surface-modified materials after torsion.

2. Theory

The model isotropic ideally elastoplastic material considered previously [1, 2] was again considered. To simplify calculations the sample was assumed to be homogeneous in the absence of any media, i.e. mechanical characteristics along every rod cross-section were supposed to be identical. The influence of active media was taken into account by introducing a surface layer of a known thickness into the sample, the layer having a macroscopic shear stress point, τ_{SS}, different from the macroscopic inner (core) shear stress point, τ_S. Interlayer boundaries were considered to have an infinitesimal thickness; medium effects on material elastic constants were neglected. The pure elastoplastic torsion of a cylindrical rod around its symmetry axis (Fig. 1) was analysed.

2.1. Homogeneous rod (inert medium test: $\tau_{SS}/\tau_S = 1$)

Using known methods and procedures of plasticity theory [3], the distribution of residual stresses can readily be found

$$\sigma_{ij}^r = 0 \quad \text{when } i,j \neq z, \varphi \tag{1a}$$

$$\tau_{z\varphi}^r(r) = \begin{cases} \tau_S - \dfrac{M_0}{I} r & \text{when } C_0 \leqslant r \leqslant b \quad (1b) \\ \left(\dfrac{\tau_S}{C_0} - \dfrac{M_0}{I}\right) r & \text{when } 0 \leqslant r \leqslant C_0 \quad (1c) \end{cases}$$

where $I = \pi b^4/2$ is a polar inertia moment of the rod cross-section, C_0 is the radius of the inner rod area which was not involved in primary plastic deformation under the external torsional moment, M_0 (the elastic core). C_0, the torsional moment, M_0, and the rod radius, b, are interrelated through the following equation

$$M_0 = \frac{2}{3}\pi b^3 \tau_S \left[1 - \frac{1}{4}\left(\frac{C_0}{b}\right)^3\right] \tag{2}$$

It follows from Equations 1 and 2 that the residual stresses at the surface of a homogeneous rod are always of different sign with respect to the outer (load) stresses caused by a torsional moment M_0. In the elastic (core) region these stresses are of the same sign. The change in sign of the residual stresses occurs at the cylindrical surface with radius $r_0 = \tau_S I/M_0$. The residual stresses reach their extreme values at the rod surface, as well as at the elastic core boundary. Fig. 2a shows the residual stresses distribution for $M_0/(\pi b^3 \tau_S) = 0.58$ (at $C_0 = 0.8b$).

It also follows that the extreme residual stresses at any value of torsional moment which is not causing a loss of bearing capacity, cannot be higher than the shear stress point. Therefore, under cyclic torsion, with moments of the same sign, the homogeneous rod will behave as if it is "autostrengthened" [3].

2.2. Surface-plasticized rod (tests in a surface active medium: $\tau_{SS}^p/\tau_S < 1$)

Let us discuss the most general case when both the surface-plasticized layer and a part of the inner zone of the rod, having an initial yield stress point, τ_S, are involved in plastic deformation under torsion. The outer (load) stresses distribution occurring in the rod due to torsional moment is found by solving the full

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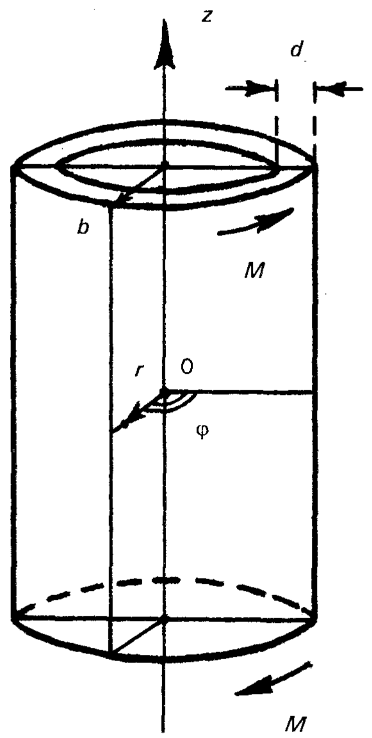

Figure 1 Structural model of a surface-modified rod in a cylindrical coordinate system: d is the thickness of a modified layer; M the torsional moment of the outer forces; b the rod radius; r, φ, z are cylindrical coordinates of an observation point.

set of equations of plasticity theory [3, 4]

$$\sigma_{ij} = 0 \quad \text{when } i, j \neq z, \varphi \tag{3a}$$

$$\tau_{z\varphi}(r) = \begin{cases} \tau_{SS}^{p} & \text{when} \quad b - d \leqslant r \leqslant b \quad (3b) \\ \tau_{S} & \text{when} \quad C_p \leqslant r \leqslant b - d \quad (3c) \\ \dfrac{\tau_S}{C_p} r & \text{when} \quad 0 \leqslant r \leqslant C_p \quad (3d) \end{cases}$$

where C_p is elastic core radius. The torsional moment M_p and C_p values are related by equation

$$M_p = \frac{2}{3}\pi b^3 \tau_S \left\{ \left(1 - \frac{d}{b}\right)^3 - \frac{1}{4}\left(\frac{C_p}{b}\right)^3 + \frac{\tau_{SS}^p}{\tau_S}\left[1 - \left(1 - \frac{d}{b}\right)^3\right] \right\} \tag{4}$$

For example, Fig. 2b shows $\tau_{z\varphi}(r)$ distribution at $d/b = 0 - 1$ and $\tau_{SS}^p/\tau_S = 0.75$ for $M_p = M_0$ (at $C_p = 0.6b$). A comparison of Equations 2 and 4 shows that if $M_p = M_0$ the depth of the layer involved in plastic deformation of a plasticized rod exceeds that of the homogeneous one. The stress tensor component, $\tau_{z\varphi}$, at the boundary between the plasticized layer and the inner zone (with $r = b - d$) undergoes a jump, the step height being equal to $\Delta\tau_{z\varphi} = \tau_S - \tau_{SS}^p$.

Residual stresses in a material under consideration can be found by solving the problem of elastic unloading in a surface plasticized rod which has been twisted to the elastoplastic state. The residual stresses distribution is as follows (see Fig. 2b)

$$\sigma_{ij}^r = 0 \quad \text{when } i, j \neq z, \varphi \tag{5a}$$

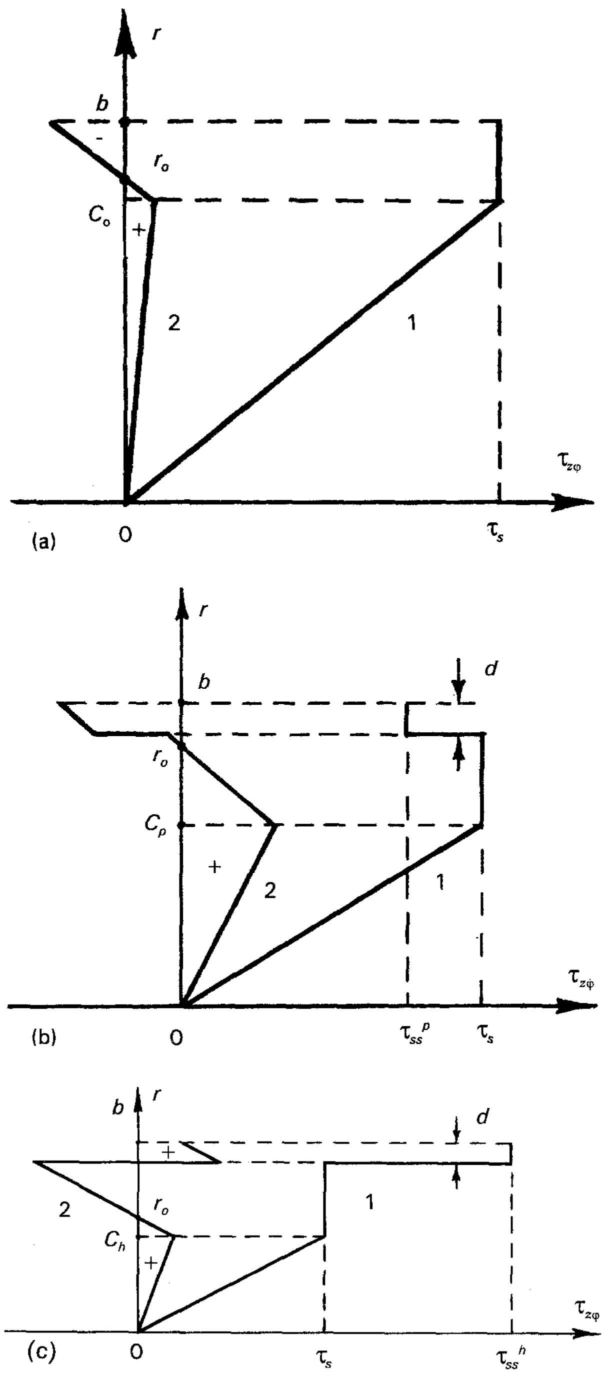

Figure 2 Distribution of (1) the outer torsional and (2) the residual stresses in (a) a uniform, (b) surface-plasticized and (c) surface-hardened rods. The outer torsional moments ratio $M_o : M_p : M_h = 1 : 1 : 1.4$.

$$\tau_{z\varphi}^r(r) = \begin{cases} \tau_{SS}^p - \dfrac{M_p}{I} r & \text{when} \quad b - d \leqslant r \leqslant b \quad (5b) \\ \tau_S - \dfrac{M_p}{I} r & \text{when} \quad C_p \leqslant r \leqslant b - d \quad (5c) \\ \left(\dfrac{\tau_S}{C_p} - \dfrac{M_p}{I}\right) r & \text{when} \quad 0 \leqslant r \leqslant C_p \quad (5d) \end{cases}$$

Comparison of Equations 1 and 5 allows us to conclude that the general character of the residual stresses distribution of homogeneous and surface-plasticized rods is qualitatively similar. In both cases the extreme values, $\tau_{z\varphi}^p$, occurred at the rod surface and in the elastic core boundary. But taken as a whole, the residual stresses level in a plasticized rod as well as gradients are considerably higher that those of a homogeneous one.

It follows from Equations 4 and 5 that if $b \gg d$ and

$$\tau_{SS}^{p}/\tau_{S} > \frac{2}{3}\left[1 - \frac{1}{4}(C_{p}/b)^{3}\right] \quad (6)$$

then the residual stresses in the external layer of the surface-plasticized rod are not higher than its shear stress point, and the secondary opposite plastic deformation of the material does not occur. Therefore, to attain the "autostrengthened" state in a surface-plasticized rod under cyclic twisting by torsional moment of the same sign, the amplitude of the torsional moment should not exceed some critical quantity. This quantity depends on the plasticized effect of the medium (i.e. on numerical values of τ_{SS}^{p}/τ_{S} and d) and for a rod of a known diameter it can be found from Equations 4 and 6. Otherwise, the fatigue strength of the rod in a plasticized medium will be markedly lower than that in an inert medium.

2.3. Surface-hardened rod (tests in a surface-active medium: $\tau_{SS}^{h}/\tau_{S} > 1$)

As in the previous version, the most general case of elastoplastic torsion was considered when both the near-surface layer and a certain part in the inner zone of the material were involved in the plastic deformation. This obviously takes place providing that $C_{h} \leqslant (b - d)\,\tau_{S}/\tau_{SS}^{h}$, where C_{h} is the elastic core radius. The relation between C_{h} and torsional moment M_{h} at $b \gg d$ within accuracy of the members $(d/b)^{2}$, is as follows

$$M_{h} = \frac{\pi b^{3}\tau_{S}}{2}\left\{\frac{4}{3} - \frac{1}{3}\left(\frac{C_{h}}{b}\right)^{3} + 4\frac{d}{b}\left(\frac{\tau_{SS}^{h}}{\tau_{S}} - 1\right)\right\} \quad (7)$$

The distribution of outer (load) stresses caused by a torsional moment, M_{h}, can be described in a surface-hardened rod as

$$\sigma_{ij} = 0 \quad \text{when } i,j \neq z,\varphi \quad (8a)$$

$$\tau_{z\varphi}(r) = \begin{cases} \tau_{SS}^{h} & \text{when } b - d \leqslant r \leqslant b \quad (8b) \\ \tau_{S} & \text{when } C_{h} \leqslant r \leqslant b - d \quad (8c) \\ \dfrac{\tau_{S}}{C_{h}} r & \text{when } 0 \leqslant r \leqslant C_{h} \quad (8d) \end{cases}$$

An example is shown in Fig. 2c by the $\tau_{z\varphi}(r)$ distribution at $d/b = 0.1$; $\tau_{SS}^{h}/\tau_{S} = 2$ and $C_{h} = (b - d)/2$.

Residual stresses can be described by the following functions (Fig. 2c)

$$\sigma_{ij}^{r} = 0 \quad \text{when } i,j \neq z,\varphi \quad (9a)$$

$$\tau_{z\varphi}^{r}(r) = \begin{cases} \tau_{SS}^{h} - \dfrac{M_{h}}{I} r & \text{when } b - d \leqslant r \leqslant b \quad (9b) \\ \tau_{S} - \dfrac{M_{h}}{I} r & \text{when } C_{h} \leqslant r \leqslant b - d \quad (9c) \\ \left(\dfrac{\tau_{S}}{C_{h}} - \dfrac{M_{h}}{I}\right) r & \text{when } 0 \leqslant r \leqslant C_{h} \quad (9d) \end{cases}$$

It follows from Equations 9 that with $\tau_{S}I/(b - d) < M_{h} < \tau_{SS}^{h}I/(b - d)$, three coaxial cylindrical zones differing in residual stresses sign exist simultaneously in the surface-hardened rod. In order to form such a structure of adjacent constricted and stretched zones in the rod with $b \gg d$, the following relationship should be satisfied

$$\tau_{SS}^{h}/\tau_{S} > \frac{4}{3}\left[1 - \frac{1}{4}\left(\frac{C_{h}}{b}\right)^{3}\right] \quad (10)$$

It can be seen that in the case of a surface-hardened rod with $\tau_{SS}^{h}/\tau_{S} > 1.33$, a three-layer structure exists at any torsional moments up to maximum one limited by the maximum bearing capacity of the rod (the latter is exhausted at $C_{h} = 0$).

Calculations prove that for the case under consideration the extreme values of the residual stresses are attained at the elastic core surface (with $r = C_{h}$) as well as at the boundary surface between the hardened external layer and the inner soft zone. In this boundary the leap (or step jump) of the stress tensor component takes place, accompanied by the alteration in sign. A repeated sign change of the residual stresses occurs in the inner zone of the rod, at the cylindrical surface with radius $r_{0} = \tau_{S}I/M_{h}$.

The analysis of the solutions also confirms the fact that maximum values of residual stresses in each zone with any actually attained torsional moments do not exceed the shear stress point of the corresponding material layers. Hence, a cyclic torsion of the surface-hardened rod by a moment of the same sign will not cause the secondary opposite plastic deformation of the material; under these conditions, "autostrengthening" of the rod occurs.

Analysis of residual stress distributions arising in the surface-plasticized and the surface-hardened rods shows that the distributions differ drastically. The main difference consists in the fact that in a plasticized rod, in contrast with the hardened one, the residual stresses in the near-surface layer always have the opposite sign with respect to the outer active stresses (caused by an outer loading moment).

It should be mentioned, however, that some common features are typical of these distributions. Among them there are oscillating changes of tangential stresses in a rod depth and the existence of high stress gradients during stress sign alteration.

3. Effects of residual stresses on defective structure of a material

The existence of adjacent layers with residual tangential stresses of opposite sign, and the higher absolute values than the dislocation start stresses, τ_{St} (see Fig. 2b and c) must intensify the contrary motion of dislocations with unidirectional Burgers vectors. Usually plastic materials are characterized by the correlation $\tau_{St}/\tau_{S} = 0.2 - 0.6$ [5].

Finally, this motion should lead to the accumulation of similar sign dislocations in cylindrical interlayer boundaries, i.e. the so-called dislocation "walls" will be formed. The equilibrium density of dislocations in the "walls" is determined by the balance of long-range stresses, σ^{a}_{ij}, caused by the own "wall" dislocations, inner stresses, σ^{in}_{ij}, originating from the stationary background dislocations (growth and net ones), and residual macroscopic stresses, σ^{r}_{ij}, existing in the material on both sides of the interlayer boundary. In the simplest case when active sliding planes in the crystal lattice coincide with coordinate planes z = constant and/or φ = constant of a base cylindrical system of axes (see Fig. 1), the balance equation allowing for solutions of the above-mentioned cases can be given as $\tau^{r}_{z\varphi} - \tau_{St}\,\tau^{r}_{z\varphi}/|\tau^{r}_{z\varphi}| = \tau^{in}_{z\varphi} + \tau^{a}_{z\varphi}$.

Using this equation and taking into account that the long-range stresses under given conditions can be caused only by screw components of "wall" and background dislocations and in so far as from [6]

$$\tau^{a}_{z\varphi} = \sum_{i=1}^{n} 0.5Ga_i(\cos\chi_i)(\rho_i)^{1/2} \quad (11)$$

and also from [7]

$$\tau^{in}_{z\varphi} = \sum_{i=1}^{n} Ga_i(\cos\chi_i)(\rho_{fi})^{1/2}/2\pi \quad (12)$$

then an expression to estimate dislocation density in interlayer boundaries can be given.

In Equations 11 and 12, G is the shear modulus, n is the number of dislocation sets forming a wall, $a_i \cos\chi_i$ is the magnitude of a screw component for Burgers vector of set i, ρ_i is the dislocation density. The stress balance equation for torsion of the sufficiently perfect samples ($\rho_i \gg \rho_{fi}$) with sliding subsystems of similar Burgers vectors ($a_i = a$) gives

$$\sum_{i=1}^{n} (\rho_i)^{1/2}\cos\chi_i = 2(\Delta\tau_{z\varphi} - \tau_{St})/Ga \quad (13a)$$

$$\rho = \sum_{i}^{n} \rho_i \quad (13b)$$

where $\Delta\tau_{z\varphi}$ is the amplitude of residual stresses near the boundaries between sample layers with different stress sign. The calculations show (Fig. 2b) that for a surface-plasticized layer, $\Delta\tau_{z\varphi} \simeq \tau_S - \tau^{p}_{SS}$, and $\Delta\tau_{z\varphi} \simeq 0.6(\tau^{h}_{SS} - \tau_S)$ for a surface-hardened layer.

Let us discuss a real example, namely a well-known test on lead wire torsion in 0.2% oleinic acid solution in vaseline oil. Monocrystalline lead wire was annealed and oriented along its load-bearing axis inclinable between nearest [1 1 0] and [1 1 1] directions of the crystalline lattice. The reciprocal location of the most active lead sliding subsystems ⟨1 1 0⟩ {1 1 1} and coordinate planes of the base cylindrical system is close enough in this case in order to use Equation 13 for estimation of the wall dislocation density. The following shear modulus and geometrical sizes are used: G = 5.5 GPa, a = 0.35 nm [7] and b = 0.5 mm [8].

Material characteristics dependent on both physicochemical conditions at the sample surface and bulk lattice defects can be estimated using experimental data [8, 9]: τ_{SS} = 2.2 MPa, $\Delta\tau_{z\varphi}$ = 1.33 MPa, τ_{St} = 0.1, $1\tau_{SS}$; d = 5–10 μm; $\rho_{fi} \simeq 10^6$ cm^{-2}. Assuming $n = 4$, χ_i = 0–35 °C and ρ_i is approximately the same for all i-set numbers and using Equation 13, the partial and total density of "wall" dislocations can be obtained: $\rho_i = (8–12)\times10^6$ cm^{-2} and $\rho \simeq 4\times10^7$ cm^{-2}.

The process of dislocations concentrating in the interlayer boundary is accompanied by the synchronous purification of near-boundary regions from the dislocations. It is easy to show that the region width is $\delta \simeq (1/\rho_i)^{1/2}$; according to the given data (for lead) $\delta \sim 0.5d$. Under these conditions (due to the small width of the surface-modified layer) an appreciable part of the layer will tend to have more ordered and regulated dislocation structure compared to that under loading. Simultaneously, the short-range effective stresses occurring in the inner boundary of the surface-modified layer due to accumulation of similar dislocations and growing progressively with the number of loading cycles (and with "proportionate" increasing of ρ, ρ_f and $\Delta\tau_{z\varphi}$) will stimulate a gradual chipping of the layer from the inner sample zone. The chipping process starts in regions of the interlayer boundary which are characterized with the values of elastic energy, γ_a, close to the free surface energy, γ_s. Assuming

$$\gamma_a = \frac{G}{4\pi}\sum_{i=1}^{n} a_i^2\rho_i^{1/2}\left[\cos^2\chi_i \ln\left(\frac{e\alpha}{2\pi a_i(\cos\chi_i)\rho_i^{1/2}}\right) + \frac{\sin^2\chi_i}{1-\nu}\ln\left(\frac{e\alpha}{2\pi a_i(\sin\chi_i)\rho_i^{1/2}}\right)\right] \quad (14)$$

where ν is Poisson's ratio, α is a parameter having a numerical value dependent on the relation between the energy of an elastic stress field of a dislocation and the energy of its core, the estimation of the critical density for "wall" dislocations in the boundary before chipping can be carried out. For example, in the tests with the lead wire at $n = 4$, χ_i = 0–35 °C and $\alpha = 4$ [6], under conditions that γ_S = 0.56 J m^{-2} [10] then $\rho_{cr} = 10^{14}–10^{15}$ cm^{-2}. Taking into account the difference in the residual stress distribution for surface-plasticized and surface-hardened samples, it can be predicted that especially intensive processes of inner-layer dislocation structure regulation, leading to the improvement of crystalline lattice, i.e. to the relative decrease of the microdistortions and to the increase of mosaic block size, will occur during pauses in the same sign cyclic torsion of the surface-plasticized rods. Indeed, residual stresses inside near-surface layers of surface-plasticized rods have an opposite sign with respect to the outer loading stresses, and during the complete cycle, the residual stresses should involve the dislocations in alternating motions which results in some part of them moving to the free sample surface and slipping away from the crystal lattice.

On the other hand, the chipping processes will predominate during the torsion of surface-hardened rods. This is because the residual tangential stresses in the external layer have the same sign as active ones, and in the interlayer boundary the residual tangential stresses drastically change in sign, promoting the additional concentration and accumulation of immobile dislocations near the boundary.

4. Conclusions

A comparative study of residual stresses arising in surface-modified material after uniaxial extension, constriction [1], bending [2] and torsion permits a general regularity typical of these cases to be deduced: in the above-mentioned tests the residual stresses arising in a plasticized surface layer of a material always have a sign opposite to that of the outer loading stresses. As a rule, in surface-hardened layers, the outer loading and residual stresses have the same sign. Hence, for a complex loading of any kind, the trajectory of which can be given by superposition of the listed plane approximations, the same regularities are still true. This concerns not only the distribution of residual stresses in near-surface layers of modified materials, but the conclusions are also universal about the influence of the active media and residual stresses on the defect structure of a material.

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